European Congress on Computational Methods in Applied Sciences and Engineering
ECCOMAS 2004
P. Neittaanmäki, T. Rossi, S. Korotov, E. Oñate, J. Périaux, and D. Knörzer (eds.)
Jyväskylä, 24—28 July 2004

ACTIVE CONTROL OF SUPPLY IN WATER NETWORKS (VIRTUAL DISTORTION METHOD APPROACH)

Nagib Nasher, Przemyslaw Kolakowski and Jan Holnicki-Szulc

SMART-TECH Centre, Institute of Fundamental Technological Research
ul. Swietokrzyska 21, 00-049 Warsaw, Poland
web page: http://smart.ippt.gov.pl/

Key words: active control, water networks.

Abstract. *The methodology for active control of supply in water networks is presented. It is based on the assumption that the water network exhibits a steady state of flow and the water heads in the network's nodes can be measured for an inspected part of the network. Making use of the graph-based model of a water network and employing the so-called Virtual Distortion Method, the problem of active control can be formulated as a constrained optimisation problem. The water supply is minimised subject to constraints on the water head in the network's nodes. The optimal solution is expected for active constraints. Analogies between the presented water network modelling and the truss modelling in structural mechanics are described.*

1 INTRODUCTION

A software tool for signal processing in control of supply in water networks is presented. It is assumed that the water pressure in the network's nodes in a distance from a controlled inlet can be measured and also that the inlet pressure can be modified in real time in a controlled way. Then, making use of the analytical network model (cf. Refs. 1, 2) of this installation and using, the so-called Virtual Distortion Method (VDM) presented below, the control of water supply can be performed.

The problem of management of water sources is more and more important on a world scale. Therefore, there is a requirement for an automatic water supply control. The proposed approach is based on continuous observation of the pressure distribution in nodes of the water network. Having a reliable (verified versus field tests) numerical model of the network and its responses for determined inlet and outlet conditions, any modifications to the normal network response (pressure distribution) can be detected. Then, applying the below proposed numerical procedure, the correction of water supply can be determined.

The proposed methodology is based on the so-called *Virtual Distortion Method* (VDM) approach, applicable also in the problem of damage identification through monitoring of piezo-generated elastic wave propagation (Ref. 3). This technique (called *Piezodiagnostics*) is focused on efficient numerical performance of inverse, non-linear, dynamic analysis. The crucial point of the concept is pre-computing of structural responses for locally generated impulse loadings by unit virtual distortions (similar to local heat impulses). These responses stored in the so-called *influence matrix* allow for a composition of all possible linear combinations of the influence of local non-linearities (due to defect) on the final structural response. Then, using a gradient-based optimization technique, the intensities of unknown, distributed virtual distortions (modelling local defects) can be tuned to minimize the distance between the computed final structural response and the measured one.

2 DEFINITIONS AND LINEAR ANALYSIS

The so-called incidence matrix L, defining topology of the network, takes the following form for the presented example:

$$L = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ -1 & 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 0 & -1 & 0 & -1 \end{bmatrix} \tag{2.1}$$

The rows of matrix L correspond to the network's nodes while its columns correspond to the branches. For instance, if you take column No. 1, responsible for the network branch No. 1, matrix L simply tells you that the branch connects nodes Nos. 1 and 2 and the <u>assumed</u> direction of the flow is from node No. 1 (value = 1) to node No. 2 (value = -1). For the remaining, unconnected nodes in the column No. 1, matrix L takes the 0 values.

Let us now define the governing equations for the water network in a steady-state flow. The equilibrium of the system relating the internal flow distribution in the network's branches Q [m^3/s] with the external inlet/outlet q [m^3/s] is expressed as follows:

$$LQ = q \tag{2.2}$$

where each equation " i " determines the balance of inlets and outlets at node " i ". The number of elements at vectors Q and q is equal to the number of nodes in the network. The analogous relation for truss structures relates internal and external forces (via the so-called geometrical matrix). The following continuity equation relates the water heads H [m] at the network's nodes with the so-called pressure heads ε [m] in the network's branches:

$$L^T H = \varepsilon \tag{2.3}$$

where each equation " j " describes flow continuity along the loop " j ". The number of elements of vectors H and ε is equal to the number of loops in? the network. The analogy for truss structures is the geometrical relation between displacements and strains. The constitutive relation for water networks relates the pressure head ε with the flow Q in the branches (strain-force relation in trusses).

$$Q^2 = R\,\varepsilon \tag{2.4}$$

The constitutive relation (2.4) is non-linear and the diagonal square matrix R (dimension equal to the number of branches in the network) is composed of the hydraulic compliance parameters R_i of each branch. R [m^2/s] is a function of the characteristic of a branch K [m^3/s] (depending upon pipe material, diameter, filtration, etc.) and its length l [m] as follows:

$$R_i = \frac{K_i^2}{l_i} \tag{2.5}$$

Nevertheless, let us temporarily assume the linearity of the relation (2.4) i.e.

$$Q = R\,\varepsilon \tag{2.6}$$

Substituting Eqs. (2.6) and (2.3) into (2.2), the following formula can be obtained:

$$LRL^T H = q \tag{2.7}$$

allowing determination of water pressure in nodal points as the response for determined inlets.

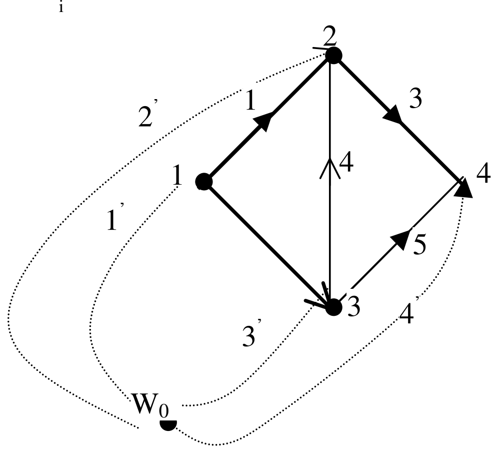
For the water network, shown in Fig.1, assuming inlet in node No.1 and outlet in node No.4, the set of equations (2.7) takes the following form:

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3+R_5 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} q_1 \\ 0 \\ 0 \\ -q_4 \end{bmatrix} \tag{2.8}$$

It was assumed that the network is supplied only through node No.1 with the inlet intensity q_1. The flow coefficients $R_i^{'}$ for fictitious branches, varying in the range between 0 and 1, can be used to determine the opening degree of the outlet valves. The intensity of the only outlet in node No. 4 can be expressed as:

$$q_4 = R_4^{'}(H_4 - H_0) \tag{2.9}$$

where the flow coefficient $R_4^{'}$ is also equal 1 (outlet valve fully open). The flow coefficients $R_2^{'}$ and $R_3^{'}$ are equal 0, which means that the outlet valves in nodes Nos. 2 and 3 are closed. Note that the inlet/outlet balance must be met (analogy to the external equilibrium condition for truss structures, which means that external loads have to be equilibrated by reaction forces in supports), that is $\sum_i q_i = 0$. In our case the condition simply yields $q_1 = q_4$.



Fictitious branches
Real branches

Fig.1 Water network

Substituting (2.9) to (2.8), assuming the reference value $H_0 = 0$ and rearranging the set of equations we can obtain the following:

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3+R_5+R_4^{'} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} q_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{2.10}$$

the solution to the above set of equations gives us pressure heads for the network supplied by the inlet q_1, and one outlet located in node No.4.

Generally, the set of equations (2.7) can be expressed (including the outlet conditions) in the following forms:

$$(LRL^T - IR') \begin{bmatrix} H \\ H^* \end{bmatrix} = \begin{bmatrix} q \\ 0 \end{bmatrix} \tag{2.11a}$$

or:

$$(L R L^T - I H^*) \begin{bmatrix} H \\ R' \end{bmatrix} = \begin{bmatrix} q \\ 0 \end{bmatrix} - L R L^T - \begin{bmatrix} 0 \\ H^* \end{bmatrix} \tag{2.11b}$$

where H* denotes water head in outlet nodes. The last equation corresponds to the case with the water head measured in the outlet nodes (and R' coefficient unknown). Contrarily, the former formula corresponds to the case with the coefficient R' measured (in the outlet node) and the water head H' unknown.

4 NON-LINEAR ANALYSIS

By analogy to the Virtual Distortion Method (VDM) applicable to the truss structures (cf.[2]) let us postulate that local modification of a network parameter can be introduced into the system through the *virtual distortion* ε^0 **,** incorporated into the formula (2.6):

$$LR(L^T H - \varepsilon^0) = q \tag{3.1}$$

The virtual distortion ε_i^0 is of the same character as the pressure head ε_i (see Fig. 2) and its physical meaning is an additional pressure head externally forced in branch " i " (e.g. due to a locally installed pump).

The influence of virtual distortions on the resultant flow redistribution can be calculated using the so-called *influence matrix* D_{ij} collecting *i* responses (row-wise) in terms of water heads $H_i^{@\varepsilon^0=1}$ induced in the network by imposing the unit virtual distortion $\varepsilon^o{}_j=1$ generated consecutively in each network branch *j*. Thus each *influence vector* $H_i^{@\varepsilon^0=1}$ can be calculated on the basis of the following equation obtained from Eq. (2.1):

$$L R L^T H^{@\varepsilon^0=1} = q^* + LRI \tag{3.2}$$

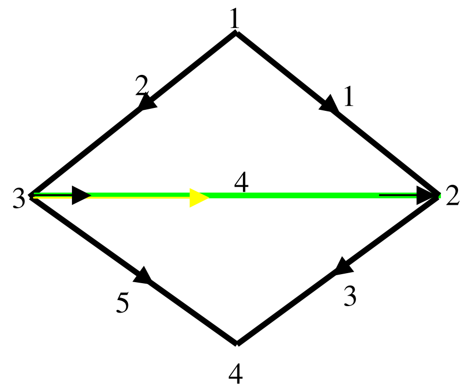


Fig. 2. Distortion simulating water flow (pressure head modification) in branch No. 4

The vector q* disregards the external inlet and outlet (the flow is now provided by the imposition of virtual distortion), and it accounts for water flow distribution in the closed network (cf. Eq. (2.8)). There is a set of j^- (j^- the number of branches) equations (2.2) to be solved in order to create the full influence matrix D. Each time the right hand-side changes as the unit virtual distortion is applied to another branch. In practice this can be realised by applying a pair of inlets-outlets $L_{ik}\, R_{kj}\, \varepsilon_j^0$ corresponding to each branch (cf. Eq. (2.1)) – it is the so-called *compensative charge*.

So the parameter modification in the system is accounted for by superposing the so-called *linear response* of the original network and the so-called *residual response* due to imposition of the virtual distortion. Therefore, the resultant water head distribution can be expressed as:

$$H_i = H_i^L + H_i^R = H_i^L + \sum_j D_{ij} \varepsilon_j^0 \tag{3.3}$$

and the resultant water flow as:

$$Q_j = Q_j^L + Q_j^R = Q_j^L + R_j L_{ij}^T \sum_j \left(D_{ij} - \delta_{ij} \right) \varepsilon_j^0 \tag{3.4}$$

Making use of the following substitution: $D^\varepsilon = L^T D$, the above relations take the form (cf. Eqs. (3.3), (3.4)):

$$\varepsilon_i = \varepsilon_i^L + \varepsilon_i^R = \varepsilon_i^L + \sum_j D_{ij}^\varepsilon \varepsilon_j^0 \tag{3.5}$$

$$Q_i = Q_i^L + Q_i^R = Q_i^L + R_i \sum_j \left(D_{ij}^\varepsilon - \delta_{ij} \right) \varepsilon_j^0 \tag{3.6}$$

Non-linearity of constitutive relations (cf. Eq. (2.4)) can be simulated through virtual distortions. To this end, let us assume that this relation is approximated through a piece-wise-linear function, for example composed of two pieces (see Fig. 2.4). The algorithm for non-linear analysis of water networks is analogous in this case to the progressive collapse analysis of elasto-plastic truss structures (cf. [2], [5]), where the sequence of overloaded (i.e. exceeding the yield stress limit due to increasing load intensity) elements should be determined and the corresponding sequence of "growing" sets of linear equations solved.

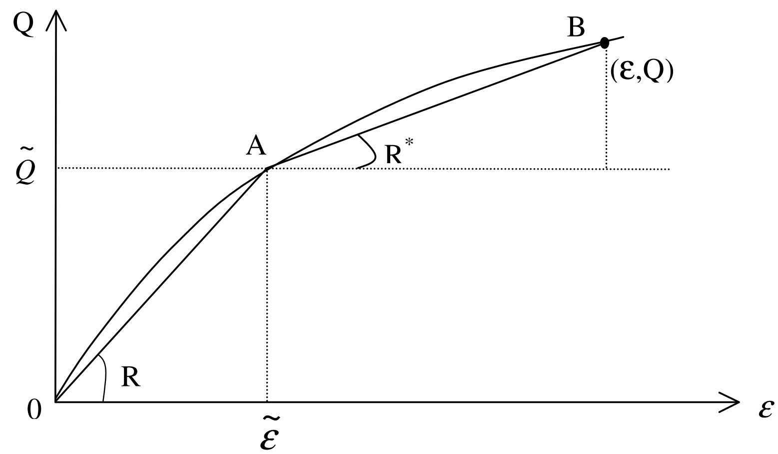


Fig. 3 Piece-wise-linear approximation of the non-linear constitutive relation

By analogy to structural mechanics, the conditions for simulation of non-linear behaviour of network branches (see Fig. 3) take the following form (describing line AB):

$$Q_i - \tilde{Q}_i = R_i^*(\varepsilon_i - \tilde{\varepsilon}_i) \tag{3.7}$$

Denoting $\gamma_i = \frac{R_i^*}{R_i}$ and substituting Eqs. (3.5), (3.6) to Eq. (3.7), the following set of linear equations is obtained:

$$\sum_l \left((1-\gamma_k)D_{kl}^{\varepsilon} - \delta_{kl}\right)\varepsilon_l^0 = -(1-\gamma_k)\left(\varepsilon_k^L - \tilde{\varepsilon}_k\right) \tag{3.8}$$

where ε_l^0 denotes virtual distortions modelling nonlinear behaviour in branch " j ".

The set (3.8) should be solved with respect to the unknown virtual distortions ε_l^0 where the indices *k, l* run through the branches of non-linear characteristics only. More accurate approximation of non-linearities requires application of more piece-wise linear sections and therefore leads to the increase of virtual distortion components ε_l^0 to be determined.

5 CONTROL OF WATER SUPPLY: LINEAR CASE

Let us discuss the *control* of water networks. The objective is to minimise the water supply (energy saving) keeping the pressure in all outlets above some limit value. Assuming that the height of outlet nodes can be monitored in real time, specially programmed controller can adapt (through a feedback procedure) the inlet intensity to meet the minimum supply condition. The aim of the following analysis is to determine the basis for the controller operation.

In the case of low pressure (below the imposed limit value) in any of the outlets, the controller provokes the increase of the inlet to achieve the right pressure level. Contrarily, in the case of pressure higher than the limit value in all outlets, the controller provokes reduction of the inlet in order to meet the limit-pressure-value in at least one outlet.

The problem of active control of the inlet intensity γq_1 (where γ denotes the controlled inlet intensity) can be formulated as follows:

$$\min \gamma \tag{4.1}$$

subject to constraints (2.11a or 2.11b) and the following conditions requiring the pressure in the outlet joints to be not smaller than some minimal admissible value H':

$$H \geq H' \tag{4.2}$$

Differentiating the Eq.2.11a with respect to γ the following set of linear equations allowing gradient determination can be provided:

$$(LRL^T - IR') \begin{bmatrix} dH \\ d\gamma \end{bmatrix} = \begin{bmatrix} q \\ 0 \end{bmatrix} \tag{4.3}$$

The solution of the linear optimisation problem formulated above with the objective function (4.1), linear constraints (2.11a or 2.11b) and the convex domain of admissible solutions (4.2) can be found on the boundary of the domain constrained by (4.2.). Calculating the? gradient (4.3) the optimal solution can be easily found (e.g. through the simplex method). Certainly, the generalised problem corresponds to the multi-inlet case, when the total inlet intensity: $\sum_i \gamma_i q_i$ has to be minimized as the objective function, rather then the scalar value (4.1). Also, the following formula for water heads has to be taken into account in this case: $H=\sum_i \gamma_i H_i$, where H_i describes the water heads in response to one inlet q_i.

Let us now discuss the above problem using the network example illustrated in Fig. 4. We assume that the coefficients R'_3 and R'_4 were previously measured and we have to calculate the unknowns: H_1 , H_2, H_3 , H_4 in order to determine outlets q_3 and q_4. In this case the corresponding set of equation looks as follows:

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5 & -R_5 \\ 0 & -R_3 & -R_5 & R_3+R_5 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} q_1 \\ 0 \\ q_3 \\ q_4 \end{bmatrix} \tag{4.4}$$

where:

$$q_3 = R_3^{'}(H_0 - H_3)$$

$$q_4 = R_4^{'}(H_0 - H_4), \qquad R = \frac{K^2}{l}, \tag{4.5}$$

and:

K- the characteristic of the element, l - the element's length,
H - the water pressure in the node (*height* of water)
q - the flow in the branch.

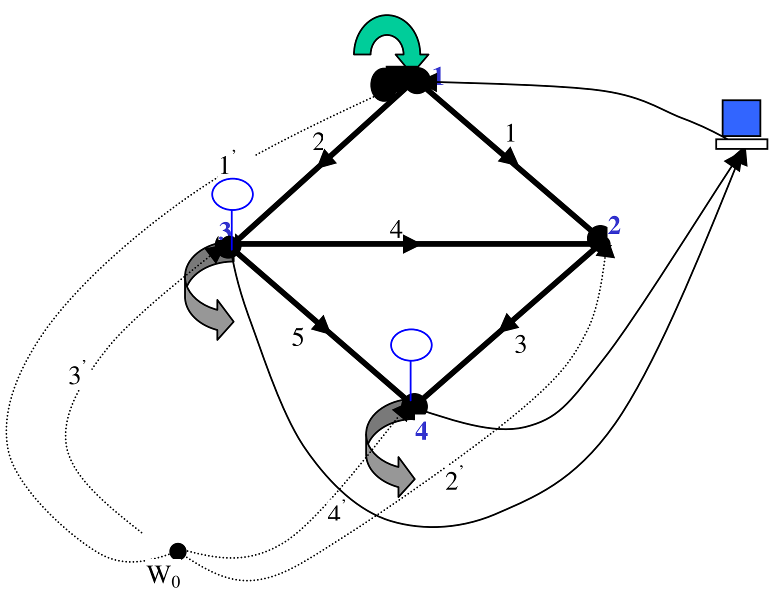


Fig. 4. The water supply control concept

Substituting Eqs. 4.5 to 4.4, the following set of equations can be derived (cf. 2.11a):

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & 0.0 \\ -R_1 & R_1+R_3+R_4 & -R_4 & -R_3 \\ -R_2 & -R_4 & R_2+R_4+R_5+R_3^{'} & -R_5 \\ 0.0 & -R_3 & -R_5 & R_3+R_5+R_4^{'} \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ H_4 \end{bmatrix} = \begin{bmatrix} q_1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{4.6}$$

where it was assumed that the network is supplied only through node No.1 (inlet with intensity q_1) and the only outlets are through nodes No.3 and No 4. $R_2^{'} = 0$, which means, that the outlet in node No. 2 vanishes. The unknowns H_1, H_2, H_3, H_4 can be determined from Eqs. 4.6 knowing the measured values of R'_3, R'_4 and q_1.

Assume the following data: $K1=K2=K3=K4=K5=0.061 m^3/s$, $l_1 = l_2 = l_3 = l_5 = 5.000$ m, $l_4=7.071$ m, $q_1=0.005\ m^3/sec$, $H_0 = 0.0$, $H'=0.00228$ and the following measurements: $R_3^{'} = R_4^{'} = 0.5$ (cf. Fig. 4). Now, we can assume that the network was monitored for a certain period of time and that the parameter $\gamma = 1.0$ corresponds to the inlet intensity for the observed, stabilised steady state flow.

Solving the set of equations (4.6) one can get H_i =[4.12680 1.52730 0.007724 0.00228], which means that the constraint $H_4 \geq H'$ is the active one. The obtained result is located, on the boundary of the admissible domain (4.2) and it is the optimal solution for the case when $\gamma = 1.0$.

The pressure head looks as follows ε^L= [2.5995 4.1191 1.5250 1.51958 0.00544] and the outlets are as follows: $q_3 = 0.00386\ m^3/s$, $q_4 = 0.00114\ m^3/s$. Note that the minimum height of outlets is $H_4 = H' = 0.00228$m and that no modification of the inlet intensity is required.

Now, let us analyze another case of the above network, when due to the outlets modification (e.g. coefficients $R_3^{'} = 0.80$, $R_4^{'} = 1.00$ are measured) water heads drops below H': $H_{min} < H'$ and an increase of the inlet intensity is required. The set of equations (4.6) leads to H = [4.1243 1.5252 0.004829 0.001137] and the corresponding pressure heads ε^L=[2.5991 4.1195 1.5241 1.5204 0.0037].

As $H_{min}=H_4 < H'$ we can expect that for the optimal solution the following condition will be satisfied: $H_4 = H'$. Then, instead of following a general, linear optimisation procedure with gradient calculation we can solve the rearranged set of equations (4.6) with respect to unknowns: H_1 , H_2 , H_3, γ:

$$\begin{bmatrix} R_1+R_2 & -R_1 & -R_2 & q_1 \\ -R_1 & R_1+R_3+R_4 & -R_4 & 0.00 \\ -R_2 & -R_4 & R_2+R_4+R_5+R_3^{'} & 0.00 \\ 0.00 & -R_3 & -R_5 & 0.00 \end{bmatrix} \begin{bmatrix} H_1 \\ H_2 \\ H_3 \\ \gamma \end{bmatrix} = \begin{bmatrix} 0.00 \\ R_3 H_4 \\ R_5 H_4 \\ -(R_3+R_5+R_4^{'}) H_4 \end{bmatrix} \tag{4.7}$$

which leads to $H_1 = 8.265$, $H_2 = 3.056$, $H_3 = 0.0097$, $\gamma = 2.0034$.

However, if more than one inlet is taken into account the gradient-based optimisation technique should be used to reach the optimal solution.

6 CONTROL OF WATER SUPPLY: NON-LINEAR CASE

The problem of active control of the inlet intensity γq_1 (where γ denotes the controlled inlet intensities) can be now generalised for the non-linear case:

$$\min \gamma \tag{5.1}$$

subject to constraints (2.11a), with distortions modelling non-linearity, and (3.8):

$$L\,R\,(L^T H - \beta^0) - I\,R'\,H = \begin{bmatrix} q \\ 0 \end{bmatrix} \tag{5.2}$$

$$R\,(L^T H - \beta^0) - \tilde{Q} = R^*(L^T H - \tilde{\varepsilon}) \tag{5.3}$$

where:

$$L^T H = L^T H^L + L^T D\,\beta^o \tag{5.4}$$

and:

$$H \geq H' \tag{5.5}$$

Substituting (5.4) to (5.2) and (5.3) one can get:

$$L\,R\,[\,L^T H^L + (L^T D - I)\,\beta^0\,] - I\,R'\,(H^L + D\,\beta^0) = \begin{bmatrix} q \\ 0 \end{bmatrix} \tag{5.6}$$

$$[R\,(L^T D - I) - R^* D]\,\beta^0 = \tilde{Q} - R^*\tilde{\varepsilon} + (R^* - R)L^T R^* \tag{5.7}$$

The set of equations (5.6) allows the determining of linear response H^L and the influence matrix D. Then, the formula (5.7) can be used to compute virtual distortions β^o modelling non-linearities in the progressive tuning of the solution.

Assuming

$$H = \gamma H^L + D\beta^o \tag{5.8}$$

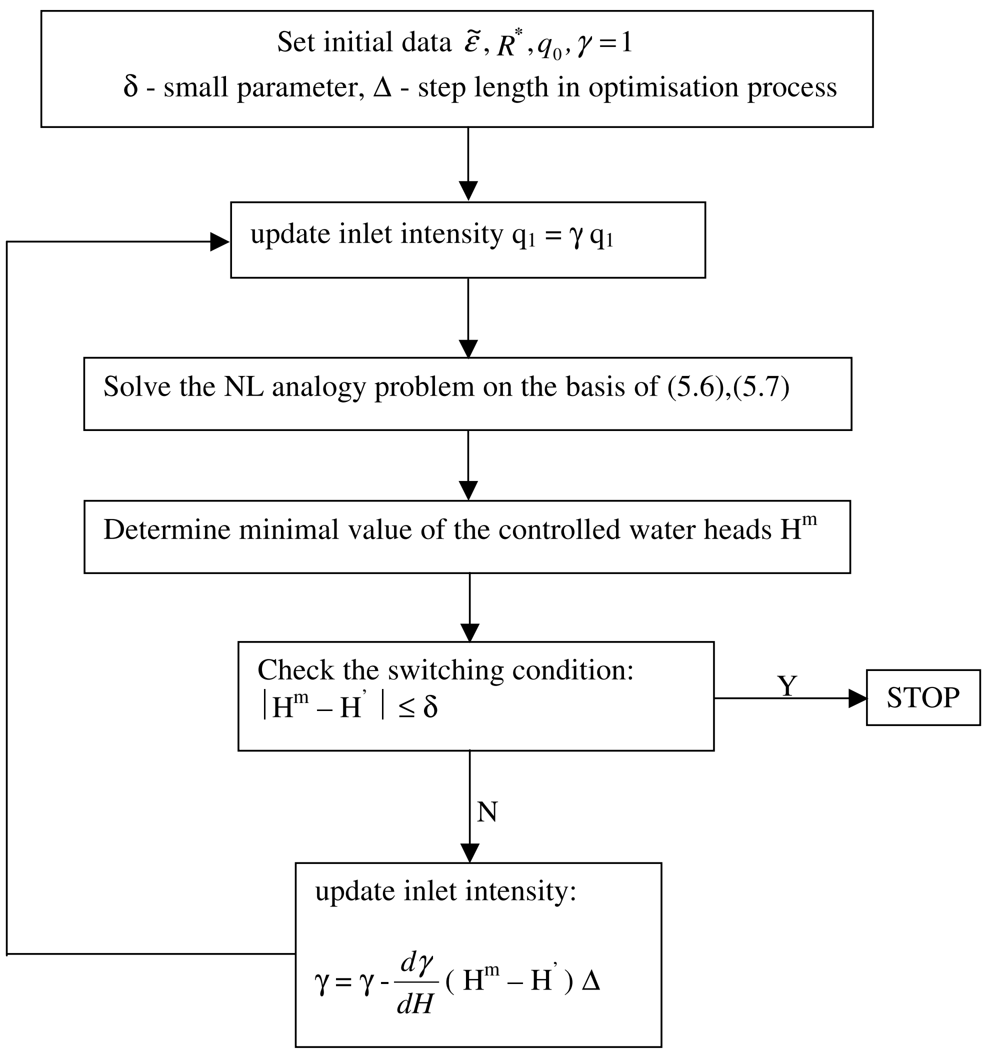
the following gradients can be calculated from (5.8) and (5.7):

$$\frac{dH}{d\gamma} = H^L + D^H \frac{d\beta^0}{d\gamma} \tag{5.9}$$

$$[\, R\,(L^T D^H - I\,) - R^* D^H \,]\ \frac{d\beta^0}{d\gamma} = (R^* - R) L^T H^L \tag{5.10}$$

The gradients (5.9), (5.10) change during the progressive analysis, when the set B of non-linear branches is modified. Calculating $d\beta^o/d\gamma$ from (5.10) and then (after substituting this result to (5.9)) $dH/d\gamma$, the gradient of the objective function can be derived. On this basis, a gradient based non-linear optimisation procedure can be used to reach the solution (cf. WATNET-C algorithm, Table 1) for which at least one of the constraints (5.5) should be active.

Table 1 WATNET-C Algorithm



Of course, the generalised problem corresponds to the multi-inlet case, when the total inlet intensity: $\sum_i \gamma_i q_i$ has to be minimized as the objective function, rather than the scalar value (5.1). Also, the following formula for water heads has to be taken into account in the above formulas in this case: $H^L=\sum_i \gamma_i H^L_i$, where H^L_i describes the water heads in response to one inlet q_i.

Let us finally demonstrate the result of active control on the basis of the example discussed above, however, with non-linear properties determined by parameters $\tilde{\varepsilon} = 3.0$ and $\alpha = 0.5$, H' = 0.0025m. Assume the following data: $K_1=K_2=K_3=K_4=K_5=0.061 m^3/s$, $l_1 = l_2 = l_3 = l_5 =$ 5.000 m, l_4=7.071 m, q_1=0.005 m^3/sec, $H_0 = 0.0$ and the following measurements: R_3' =0.80, R_4' = 1.0, $\alpha = 0.5$ and $\bar{\varepsilon} = 3.0$).

First, we have to calculate the influence matrix D, H^L and ε^L from (5.6) and then the value of distortions modelling the non-linearity from Eq. (5.7) which leads to β^o = [0.000 0.807343 0.000 0.000 0.000].

The resultant water heads calculated from (5.8) take the following form H = [4.6193 1.7081 0.00466 0.00127] and the corresponding pressure head looks as follows ε = [2.9113 4.6147 1.7068 1.7034 0.0039] .

From this solution we can see that the water head H_4 is smaller than H' which means that the inlet has to be increased in order to achieve the right level. Following the gradient-based algorithm (Table 1) the optimal solution can be reached (for γ=2.2) β^o = [2.7774 4.6598 0.3858 0.3824 0.000] and H = [12.3289 3.7741 0.0093 0.0025].

As long as the inlet is limited to one node the optimisation process is trivial and limited to only one control parameter. However, having multi-inlet water network, which is the case in the majority of real large installations, the algorithm described in Table 1 with gradient-based optimisation approach (for several control parameters) is required.

ACKNOWLEDGEMENTS

The authors would like to gratefully acknowledge the financial support through the FP5 Research Training Networks Project HPRN-CT-2002-00284 (2002-2006) "SMART SYSTEMS - New Materials, Adaptive Systems and Their Nonlinearities: Modelling, Control and Numerical Simulation" and through the grant No. KBN 5 T07A 052 22, funded by the State Committee for Scientific Research in Poland.

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